Lecture 19

Wednesday, February 24, 2021 2:15 PM

* Prayer
* Spiritual thought
* Answering questions

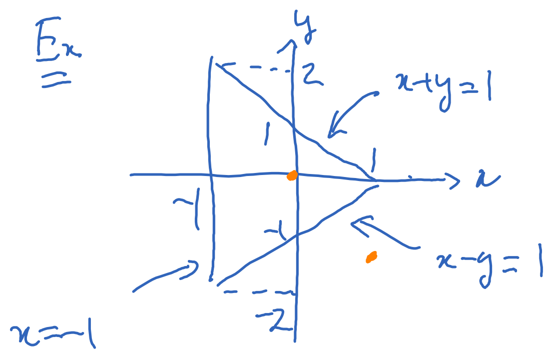
Find absolute extrema of a function.

$$f(x,y,\ldots) \to \max/\min \quad \text{on } D$$

Rule of thumb:

[If max/min exists, it must occur on the boundary of D or at a critical point inside D.]

Ex



$f(x,y) = 3x^2 - x^3 + 2xy + y^2$

Two critical points: $(0,0)$ and $\left(\frac{4}{3}, -\frac{4}{3}\right)$.

On the edge $x=-1$: .... $f(x,y) = 3+1-2y+y^2 = y^2-2y+4$

min = 3, max = 7

On the edge $x+y=1$: ....

$$f(x,y) = 3x^2 - x^3 + 2x(1-x) + (1-x)^2$$

$$= -x^3 + 2x^2 + 1 \to \min = 1, \ \max = 3$$

$-3x^2 + 4x = x(\underbrace{4-3x}_{\geq 0})$

On the edge $x-y=1$:

$$f(x,y) = 3x^2 - x^3 + 2x(x-1) + (x-1)^2$$
$$= \underbrace{-x^3 + 6x^2 - 4x + 1}_{h(x)}$$

$h(4-\frac{2}{3}\sqrt{6}) \approx 11.87$

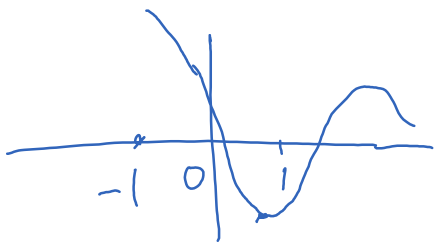
$h(-1) = 12$

$h'(x) = -3x^2 + 12x - 4$

$\Delta' = 36 - 12 = 24$

$x = \dfrac{-12 \pm \sqrt{24}}{-3}$

$= 4 \pm \frac{2}{3}\sqrt{6}$



$3x^2 - 12x + 4$

$h(-1) > 0 \quad h(0) > 0 \quad h(1) < 0$

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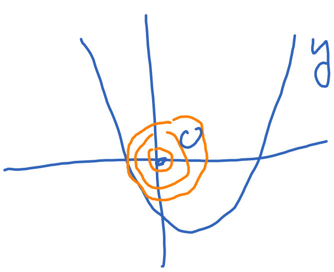
* Extrema under contraints

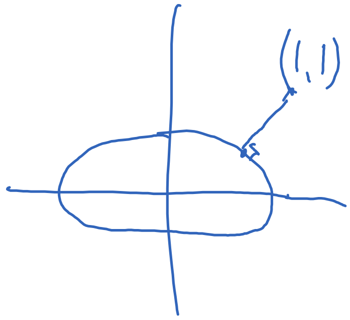


$y = 4x^2 + 2x + 1$

Find the point on the parabola that is closest to the origin.

$$\begin{cases} x^2 + y^2 \longrightarrow \min \\ y - 4x^2 - 2x = 1 \end{cases}$$

Ex



$$\overbrace{x^2 + 4y^2}^{g(x,y)} = 1$$

$$\underbrace{(x-1)^2 + (y-1)^2}_{f(x,y)} \to \min$$

Rule of thumb:

$$\begin{cases} f(x,y,\ldots) \to \min/\max \\ g(x,y,\ldots) = k \quad (\text{fixed}) \\ (x,y,\ldots) \in D \end{cases}$$

min/max occurs either on the boundary of D or where $(x,y,\ldots)$ solves the system

$$\begin{cases} \nabla f(x,y,\ldots) = \lambda \nabla g(x,y\ldots) \\ g(x,y,\ldots) = k \end{cases}$$